Modern Terrestrial Reference Systems (Part 1)

Dr. Richard A. Snay and Dr. Tomás Soler

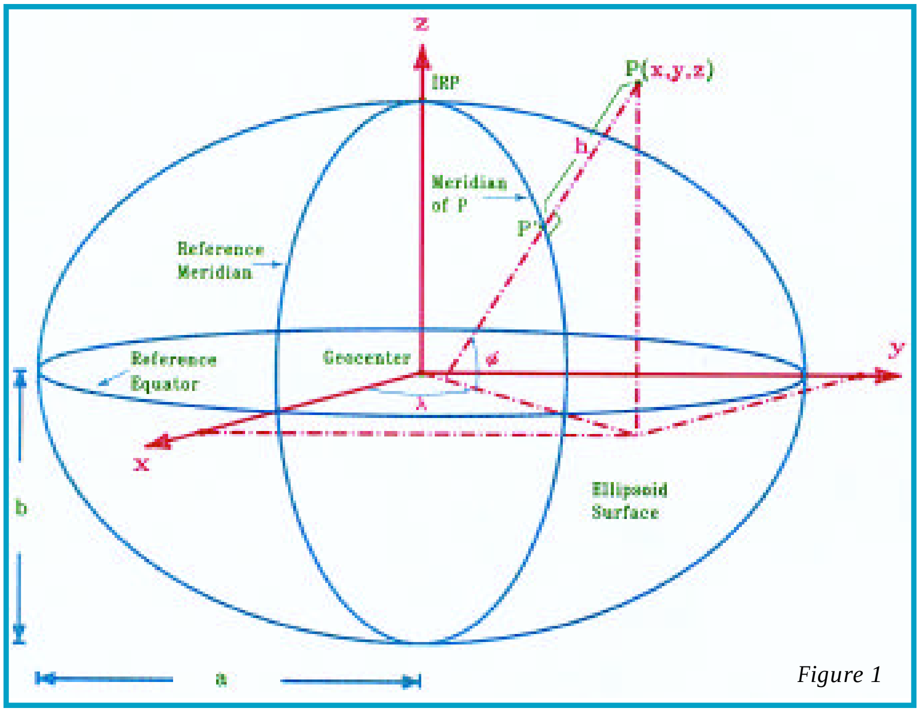

Figure 1

Surveyors, GIS/LIS professionals, engineers, cartographers, and others who work in North America face the challenge of dealing with at least three different 3D terrestrial reference systems. For many legal activities, these people express positional coordinates in the reference system known as the North American Datum of 1983 (NAD 83). Alternatively, they often favor using the World Geodetic System of 1984 (WGS 84) for various practical positioning activities involving the Global Positioning System (GPS), or they find the International Terrestrial Reference System (ITRS) more suitable for achieving superior positional accuracy. While these three reference systems differ from one another only slightly in concept, they differ significantly in how they have been realized, where the realization of a particular reference system is called a "reference frame". A particular reference frame is usually established by designating positions and velocities for several identifiable points. To date there have been several realizations of each of these three reference systems, as institutions have systematically revised positions and velocities from time to time to keep pace with how evolving technology has improved positioning accuracy. Here, we review the evolution of these reference frames, and we discuss transforming positions between different reference frames. Finally, we address some practical considerations for accurate positioning and discuss plans for a new NAD 83 realization.

Defining a Reference System

The modern approach to defining a 3D terrestrial reference system may be divided into four steps. The first step links the axes of a 3D cartesian coordinate system to a configuration of physically measurable locations on or within the earth. As a result, the location and orientation of the three coordinate axes are defined. The second step relates the concept of distance to physically measurable quantities whereby a unit of length is introduced. The third step introduces an auxiliary geometric surface that approximates the size and shape of the earth. Finally, the fourth step addresses the question of how Earth's gravity field contributes to the notion of position, and especially that of height. We shall be concerned here with only the first three steps, thus focusing on the geometric aspects involved in defining a reference system.

For the first step, most scientists involved in defining modern reference systems agree that the origin of the 3D cartesian system should be located at Earth's center of mass (geocenter); also that the cartesian system's z-axis should pass through the conventional definition of the North Pole, or more precisely, the International Reference Pole (IRP) as defined by the International Earth Rotation Service (IERS), an international organization established in 1988 and headquartered in Paris, France. The x-axis should go through the point of zero longitude located on the plane of the conventional equator, which is also defined by the IERS. The meridian going through this point is located very close to the meridian of Greenwich although the two are not coincident. The y-axis forms a right-handed coordinate frame with the x- and z-axes. Indeed, each of the three reference systems—NAD 83, WGS 84, and ITRS—has been accordingly defined in concept. They differ, however, as we shall soon discuss, in their realizations; that is, in how the location and orientation of their respective cartesian axes have been physically materialized as well as their respective concepts of distance. Unfortunately, what initially appears to be a simple geometric procedure is complicated by Earth's dynamic behavior. For example, Earth's center of mass is moving relative to Earth's surface. Also, there are variations of Earth's rotation rate as well as motions of Earth's rotation axis both with respect to space (precession and nutation) and to Earth's surface (polar motion). Moreover, points on the earth's crust are moving relative to one another as a result of plate tectonics, earthquakes, volcanic/magmatic activity, postglacial rebound, people's extraction of underground fluids, solid Earth tides, ocean loading, and several other geophysical phenomena. Modern terrestrial reference systems, hence, need to account for these motions. One option is to relate the cartesian axes to the locations of selected points measured at a particular instant of time (epoch). This alternative is generally used when dealing with the motion of the earth's rotational axis and with the motions associated with plate tectonics. Other types of motion (for example, subsidence) are accounted for by fixing the cartesian axes to some temporal average of the locations for selected points. As we shall see, a fundamental difference among the various reference frames involves how they address the motion associated with plate tectonics.

For the second step, scientists concerned with defining up-to-date terrestrial reference systems agree that the unit of length, the meter or "metre", corresponds to the length of path traveled by light in a vacuum during a time interval of exactly 1/299,792,458 seconds. This solves the problem of relating the concept of distance to a physically measurable quantity in theory, but not in realization. Each of the various reference frames associated with NAD 83, WGS 84, and ITRS relies on a distinct set of measurements that were performed using one or more of several widely different types of instruments and techniques, among the most representative: GPS, electro-optical distance measuring instrumentation, Doppler satellite positioning, very long baseline interferometry (VLBI), and satellite laser ranging (SLR). While each measurement type had been calibrated to fit the definition of a meter as best as possible, the observations, nevertheless, contain uncertainties. Consequently, the "scale" of any particular reference frame is somewhat less than perfect. In particular, when old classical terrestrial frames are compared with modern "space-age" frames, scale errors at the part-per-million (ppm) level may often be detected. Because of recent technological advances in the measurement of time and, consequently, distance, scale differences between modern frames are now approaching the part-per-billion (ppb) level.

For the third step, the earth's surface is approximated in size and shape by the geometric surface that is formed by rotating an ellipse about its smaller axis (Figure 1). The generated surface is termed an "ellipsoid of revolution" or simply ellipsoid. The ellipsoid's geometric center should be located at the origin of the 3D cartesian system, and its axis of radial symmetry (semi-minor axis) should coincide with the cartesian z-axis of the selected terrestrial reference frame. The size and shape of the rotated ellipse may be completely specified using two parameters: the length of its semi-major axis, usually denoted a, which approximates the distance from the geocenter to a point on the equator (approximately 6,378 km); and the length of the semi-minor axis, denoted b, which approximates the distance from the geocenter to the North Pole. The value of b is about 0.3% shorter than a. The fact that a is longer than b is a consequence of the force imparted by Earth's rotation causing our home planet to bulge outward around its equator. Often, instead of b, the ellipsoid's flattening, $f = (a - b)/a$, is used.

Different ellipsoids have been adopted for the different reference systems. NAD 83 uses the same ellipsoid as the Geodetic Reference System of 1980 which was adopted by the International Association of Geodesy. WGS 84 uses an ellipsoid adopted by the National Imagery and Mapping Agency (NIMA, formerly the Defense Mapping Agency), and ITRS uses an ellipsoid adopted by the IERS. Corresponding values of a and f are presented in the table below.

Given values for a and b (or f), people can convert 3D cartesian coordinates—x, y, z— into the curvilinear coordinates—latitude, longitude, and ellipsoidal height—and vice versa. These curvilinear coordinates embody a certain intuitive appeal in specifying locations on and near the earth's surface, as they relate to our innate sense of the horizontal and vertical dimensions.

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Reference System	Semi-major axis, m	Flattening, unitless
NAD 83	6,378,137.0	1/298.257222101
WGS 84	6,378,137.0	1/298.257223563
ITRS	6,378,136.49	1/298.25645

Modern Terrestrial Reference Systems
PART 2: The Evolution of NAD 83

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The first realization of NAD 83 was introduced in 1986 by a group of institutions representing the various North American countries to upgrade the previous reference system; that is, the North American Datum of 1927 or NAD 27. In particular, the National Geodetic Survey (NGS) represented the United States, and this federal institution officially refers to the first NAD 83 realization as NAD 83 (1986). For this realization, the group of institutions relied heavily on Doppler satellite observations collected at a few hundred sites to estimate the location of the Earth's center of mass and the orientation of the 3D cartesian axes. They also relied on these same Doppler observations to provide scale for NAD 83 (1986). More precisely, the group of institutions relied on 3D Doppler-derived positions that had been transformed by:

- a translation of 4.5 m along the z-axis
- a clockwise rotation of 0.814 arc sec onds about the z-axis
- a scale change of -0.6 ppm

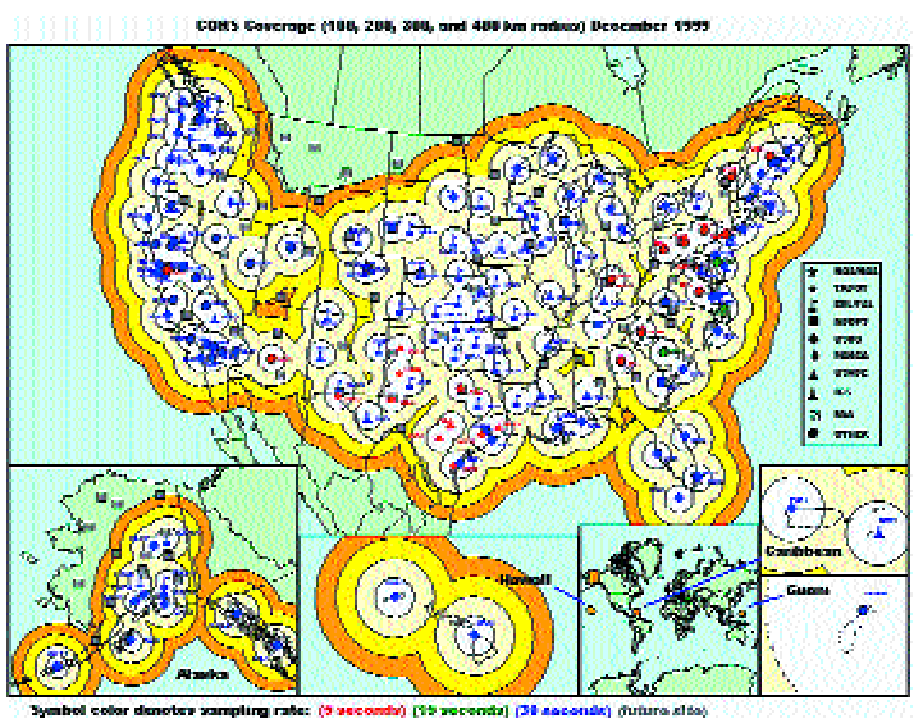
The Doppler-derived positions were so transformed to make them more consistent with the very long baseline interferometry (VLBI), satellite laser ranging (SLR), and terrestrial azimuth measurements that were available in the early 1980s. While NAD 83 (1986) is 3D in scope, NGS adopted only horizontal coordinates (latitude and longitude) for over 99% of the approximately 250,000 U.S. control points that were involved in defining this reference frame. Unfortunately, this first realization of NAD 83 occurred a few years before GPS technology made the vertical dimension economically accessible.

GPS Changed Everything

Around the same time that NGS adopted NAD 83 (1986), the agency had begun using GPS technology, instead of triangulation and/or trilateration, for horizontal positioning. The fact that GPS technology also provided accurate ellipsoidal heights was somewhat overlooked in the 1980s because surveyors, hydrologists, and other users of vertical positions required orthometric heights relative to mean sea level, as obtained with tide gauges and spirit leveling, and not geometric heights relative to an abstract mathematical surface (the ellipsoid), as obtained with GPS. The attitude towards using GPS to measure heights gradually evolved, however, as NGS and other institutions developed improved geoidal models for determining the spatial separation between mean sea level and the ellipsoid. These improvements enabled people to convert ellipsoidal heights into orthometric heights with greater and greater accuracy. Moreover, practitioners can measure heights much more economically with GPS than with spirit leveling.

As GPS matured, so did other space-age geodetic technologies; in particular, SLR and VLBI. Within a few years after 1986, both GPS and SLR measurements had allowed geodesists to locate Earth's center of mass with a precision of a few centimeters. In doing so, these technologies revealed that the center of mass that was adopted for NAD 83 (1986) is displaced by about 2 m from the true geocenter. Similarly, GPS, SLR, and VLBI revealed that the orientation of the NAD 83 (1986) cartesian axes is misaligned by over 0.03 arc seconds relative to their true orientation, and that the NAD 83 (1986) scale differs by about 0.0871 ppm from the true definition of a meter. These discrepancies caused significant concern as the use of highly accurate GPS measurements proliferated. In particular, starting with Tennessee in 1989, each state–in collaboration with NGS and various other institutions–used GPS technology to establish regional reference frames that were to be consistent with NAD 83. The corresponding networks of GPS control points were originally called High Precision Geodetic Networks (HPGN). Currently, they are referred to as High Accuracy Reference Networks (HARN). This latter name reflects the fact that relative accuracies among HARN control points are better than 1 ppm, whereas relative accuracies among pre-existing control points were nominally only 10 ppm.

For defining these regional reference frames, NGS retained the location of the geocenter and the orientation of the 3D cartesian axes which had been derived in 1986 from the transformed Doppler observations. This agency, however, opted to introduce a new scale that would be consistent with the scale of the then cur-



rent ITRS realization which is known as the International Terrestrial Reference Frame of 1989 (ITRF89). As we shall discuss later, the ITRF89 scale was based on a combination of GPS, SLR, VLBI, and lunar-laser-ranging (LLR) measurements. The resulting scale change, equal to -0.0871 ppm, altered existing NAD 83 latitudes and longitudes insignificantly, but it systematically decreased all ellipsoidal heights by about 0.6 m (= 0.0871•10-6 • R where R is the radius of the Earth). Nevertheless, this change to a more accurate scale facilitated the migration toward using GPS technology for deriving accurate heights. Let us call this second realization NAD 83 (HPGN) or NAD 83 (HARN); but keep in mind that this second realization is actually a collection of regional realizations that were formulated over a period of several years (1989-1997) with each new regional realization being "adjusted" to fit with those that preceded it.

Third Realization of NAD 83 Incorporated CORS

In late 1994, NGS introduced a third realization of NAD 83 when the agency organized a network of continuously operating reference stations (CORS). Each CORS includes a GPS receiver whose data NGS collects, processes, and disseminates for public use. Surveyors and other professionals can apply CORS data to position points at which other GPS data have been collected with accuracies that approach a few centimeters, both horizontally and vertically. The CORS system started with about a dozen sites in December 1994, and it has grown at a rate of about three sites per month. Figure 2 depicts the recent status of the National CORS network. Positional coordinates of the early CORS sites were first computed in the ITRS realization known as ITRF93. Equivalent NAD 83 coordinates were then computed by applying a Helmert transformation; that is, a transformation of the form

$$x_{NAD83} = T_x + (1+s) \cdot x_{ITRF} + R_z \cdot y_{ITRF} - R_y \cdot z_{ITRF} \quad (1a)$$

$$y_{NAD83} = T_y - R_z \cdot x_{ITRF} + (1+s) \cdot y_{ITRF} + R_x \cdot z_{ITRF} \quad (1b)$$

$$z_{NAD83} = T_z + R_y \cdot x_{ITRF} - R_x \cdot y_{ITRF} + (1+s) \cdot z_{ITRF} \quad (1c)$$

where Tx, Ty, and Tz represent three translation along the x-axis, y-axis, and z-axis, respectively, which will bring the origin of the two frames into coincidence. Rx, Ry, and Rz represent three rotations about the x-axis, y-axis, and z-axis, respectively, which, in combination, will bring the three axes of one frame into parallel alignment with their corresponding axes in the other frame. Finally, s represents the difference in scale between the two frames. The values of Tx, Ty, Tz, Rx, Ry, and Rz had been estimated so that the ITRF93 positional coordinates of nine VLBI sites in the United States would transform as best as possible (in a least squares sense) to their adopted NAD 83 (HARN) positional coordinates. The scale difference, s, was set to equal zero. These VLBI sites were used because they had highly accurate positions (cm-level) in both ITRF93 and NAD 83 (HARN). Let us use the label NAD 83 (CORS93) to identify the reference frame obtained by applying this transformation to convert CORS positions from ITFR93 to NAD 83.

In the spring of 1996, NGS computed positional coordinates for all the then existing CORS in yet another ITRS realization, known as ITRF94. Similarly, the agency developed a Helmert transformation from ITRF94 to NAD 83 using eight of the same VLBI sites (a VLBI site in California was not used here because of crustal motion concerns). Again, the scale difference was set to equal zero. NGS applied this new transformation to convert ITRF94 coordinates for the CORS sites to a fourth NAD 83 realization which we call NAD 83 (CORS94).

Most recently in the fall of 1998, NGS computed positional coordinates for all the then existing CORS in the ITRS realization known as ITRF96. This time, however, NGS collaborated with representatives from Canada's Geodetic Survey Division to derive a Helmert transformation based on eight VLBI sites in the United States and four VLBI sites in Canada. Again, a scale difference equal to zero was enforced. Other adopted parameters for this transformation are presented in the sidebar box. NGS applied the resulting transformation to convert ITRF96 positional coordinates for the CORS sites to a fifth NAD 83 realization which we call NAD 83 (CORS96). NGS, however, continues to use NAD 83 (CORS94) positions for all CORS sites, except those that have come online since the fall of 1998 and those whose NAD 83 (CORS96) position differs from their corresponding NAD 83 (CORS94) position by more than 2 cm horizontally or 4 cm vertically.

In summary, surveyors and others have witnessed five realizations of NAD 83 in the United States. A similar evolution occurred in Canada, but the two countries at least agree on their first and last realizations. The five U.S. realizations are consistent in their choice of origin and orientation; they differ, however, in their choice of scale. While the scale difference between NAD 83 (1986) and NAD 83 (HARN) equals -0.0871 ppm, the scale difference between NAD 83 (HARN) and any NAD 83 (CORSxx) is smaller than 0.005 ppm in magnitude. It should be noted that the NAD 83 (HARN) latitude and/or longitude of a given control point may differ by up to a meter from its corresponding NAD 83 (1986) coordinate. Fortunately, the horizontal discrepancy between the NAD 83 (CORS93) and NAD 83 (HARN) positions for a control point is almost always less than 10 cm, and the horizontal discrepancy between any two NAD 83 (CORSxx) positions for a control point is almost always less than 2 cm. In addition, as NAD 83 has evolved from mostly a horizontal reference system to a full 3D reference system, the number of control points with measured ellipsoidal heights has grown dramatically.

Finally, for the present discussion, we intentionally omitted the role of crustal motion on the evolution of NAD 83. We will address this topic in some detail, however, after discussing the evolution of ITRS, as this international reference system provides the global perspective that will help us understand certain concepts associated with crustal motion.

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Modern Terrestrial Reference Systems
PART 3: WGS 84 and ITRS

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The Department of Defense (DoD) developed the WGS 84 reference system to support global activities involving mapping, charting, positioning, and navigation. More specifically, DoD introduced WGS 84 to express satellite orbits; that is, satellite positions as a function of time. Accordingly, WGS 84 is widely used for "absolute" positioning activities whereby people assume that satellite orbits are sufficiently accurate to serve as the sole source of control for positioning points of interest. In particular, absolute positioning does not rely on using positional coordinates for pre-existing terrestrial points for control, except indirectly in that orbits are derived from adopted positions for a small set of tracking stations (Fig. 3). The general user, however, never needs to know the positions of these tracking stations.

DoD provides both "predicted" and "postfit" orbits in the WGS 84 reference system. As implied by the name, predicted orbits are calculated ahead of time by applying physical principles to extrapolate currently observed satellite positions. On the other hand, postfit orbits are calculated from previously observed satellite positions. Postfit orbits are more precise than predicted orbits both because they do not involve predicting the future and because they are usually derived using a larger number of tracking stations. GPS predicted orbits and satellite clock parameters are generated by the Air Force at the GPS Operational Control Segment, located at Schriever AFB, Colorado. The Air Force then uploads these predicted quantities to the GPS satellites so that this information may be included in the radio signal transmitted by these satellites. These predicted orbits support all real-time positioning and navigation activities involving GPS. Postfit GPS orbits and satellite clock parameters are generated by the National Imagery and Mapping Agency (NIMA), who currently makes this information available on its Geodesy and Geophysics World Wide Web pages. A number of other organizations also generate postfit GPS orbits which they usually express in a particular realization of the International Terrestrial Reference System (ITRS).

The original WGS 84 realization essentially agrees with NAD 83 (1986). Subsequent WGS 84 realizations, however, approximate certain ITRS realizations. Because GPS satellites broadcast the predicted WGS 84 orbits, people who use this broadcast information for positioning points automatically obtain coordinates that are consistent with WGS 84. Hence, the popularity of using GPS for real-time positioning has promoted greater use of WGS 84. Despite its popularity, people generally do not use WGS 84 for high-precision positioning activities, because such activities require the use of highly accurate positions on pre-existing terrestrial points for control. For example, various differential GPS techniques use known positions for one or more pre-existing terrestrial points to remove certain systematic errors in computing highly precise positions for new points. Consequently, before WGS 84 can support high-precision positioning activities, a rather extensive network of accurately positioned WGS 84 terrestrial control points would have to be established.

DoD established the original WGS 84 reference frame in 1987 using Doppler observations from the Navy Navigation Satellite System (NNSS) or TRANSIT. The WGS 84 frames have evolved significantly since the mid-1980s. In 1994, DoD introduced a realization of WGS 84 that is based completely on GPS observations, instead of Doppler observations. This new realization is officially known as WGS 84 (G730) where the letter G stands for "GPS" and "730" denotes the GPS week number (starting at 0h UTC, 2 January 1994) when NIMA started expressing their derived GPS orbits in this frame. The latest WGS 84 realization, called WGS 84 (G873), is also based completely on GPS observations. Again, the letter G reflects this fact, and "873" refers to the GPS week number starting at 0h UTC, 29 September 1996. Although NIMA started computing GPS orbits in this frame on this date, the GPS Operational Control Segment did not adopt WGS 84 (G873) until 29 January 1997.

The origin, orientation, and scale of WGS 84 (G873) are determined relative to adopted positional coordinates for 15 GPS tracking stations: five of them are maintained by the Air Force and ten by NIMA (see Fig. 3). NIMA chose their sites to complement the somewhat equatorial distribution of the Air Force sites and to optimize multiple station visibility from each GPS satellite. People may anticipate further improvements of WGS 84 in the future, as new GPS tracking sites may be added or existing antennas may be relocated or replaced. NIMA is dedicated to take appropriate measures to guarantee the highest possible degree of quality and to perpetuate the accuracy of WGS 84. As mentioned earlier, however, most regions lack a network of accessible reference points that might serve as control points from which highly accurate WGS 84 coordinates may be propagated using an appropriate static differential GPS technique involving carrier phase observables. Another minor drawback affecting accurate GPS work is the unavailability to the general GPS user of the crustal velocities at the WGS 84 tracking stations. More information about WGS 84 may be obtained via the Internet by accessing: http://164.214.2.59/GandG/tr8350_2.html

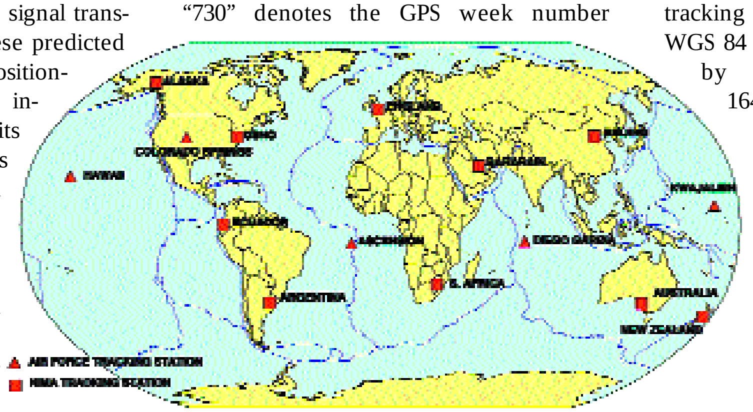

Figure 3. Combined DoD tracking network that defines WGS 84.

The Evolution of ITRS

In the late 1980s, the International Earth Rotation Service (IERS) introduced ITRS to support those scientific activities that require highly accurate positional coordinates; for example, monitoring crustal motion and the motion of Earth's rotational axis. The initial ITRS realization was called the International Terrestrial Reference Frame of 1988 (ITRF88). Accordingly, IERS published positions and velocities for a worldwide network of several hundred stations. The IERS, with the help of several cooperating institutions, derived these positions and velocities using various highly precise geodetic techniques including GPS, VLBI, SLR, LLR, and DORIS (Doppler orbitography and radiopositioning integrated by satellite). Every year or so since introducing ITRF88, the IERS has developed a new ITRS realization –ITRF89, ITRF90, ..., ITRF97– whereby they have published revised positions and velocities for previously existing sites, as well as new positions and velocities for those sites that had been established since after earlier realizations had been developed. Each new realization not only incorporated at least an additional year of data, but also the most current understanding of Earth's dynamic behavior. The ITRF96 frame is defined by the positions and velocities of 508 stations dispersed among 290 globally distributed sites (Fig. 4). Recall that a particular site may involve one or more co-located instruments employing various space-related techniques (e.g., GPS, VLBI, SLR, LLR, and DORIS). The accuracy and rigor of ITRS has proven contagious, and its popularity is steadily growing among those who engage in positioning activities.

Furthermore, ITRS is the first major international reference system to directly address plate tectonics and other forms of crustal motion by publishing velocities as well as positions for its control points. To appreciate the need for velocities, consider the theory of plate tectonics. According to this theory, Earth's outer shell consists of about 20 plates that are essentially rigid, and these plates move mostly laterally relative to one another like several large sheets of ice on a body of water. The relative motion between points on different plates are, in some cases, as large as 150 mm/yr, which is easily detectable using GPS and other modern day positioning techniques.

Given the fact that each tectonic plate is moving relative to the others, one may ask how crustal velocities may be expressed in "absolute" terms. The people responsible for ITRS currently address this

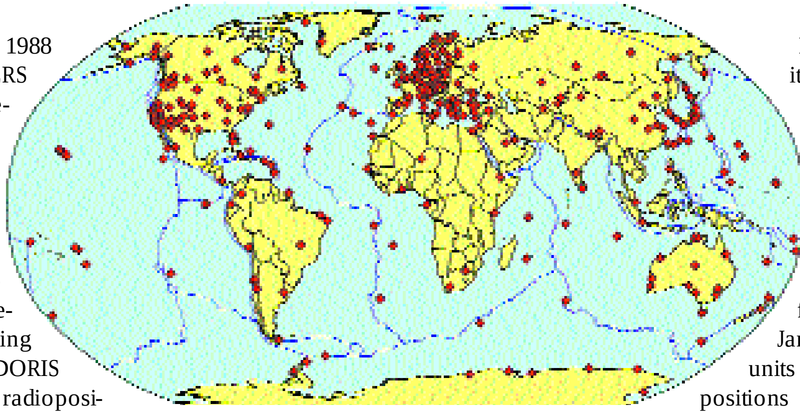

Figure 4. Sites defining ITRF96.

dilemma by assuming that the Earth's surface, as a whole, does not move "on average" relative to Earth's interior. Said differently, the ITRS developers assume that the total angular momentum of Earth's outer shell is zero. Hence, the angular momentum associated with the motion of any one plate is compensated by the combined angular momentum associated with the motions of the remaining plates. Consequently, points on the North American plate generally move horizontally at measurable rates according to the ITRS definition of absolute motion. In particular, horizontal ITRF96 velocities have magnitudes between 10 and 20 mm/yr in the conterminous 48 states. Moreover, horizontal ITRF96 velocities have even greater magnitudes in Alaska and Hawaii.

In contrast, the NAD 83 reference system addresses plate motion under the assumption that the North American plate, as a whole, does not move "on average" relative to Earth's interior. Hence, points on the North American plate generally have no horizontal velocity relative to NAD 83 unless they are located near the plate's margin (California, Oregon, Washington, and Alaska) and/or they are affected by some other deformational process (volcanic/magmatic activity, postglacial rebound, etc.). The NAD 83 reference system, however, does make special accommodations for certain U.S. regions that are located completely on another plate. In Hawaii, for example, NAD 83 positional coordinates are defined as if the Pacific plate is not moving. This approach is convenient for people who are involved with positioning activities solely in Hawaii. This approach, however, introduces a layer of complexity for people who are involved in positioning points in Hawaii relative to points in North America.

In the realm of crustal motion, it is inappropriate to specify positional coordinates without specifying the "epoch date" for these coordinates; that is, the date to which these coordinates correspond. Accordingly, ITRF96 positions are usually specified for the epoch date of 1 January 1997 (often denoted in units of years as 1997.0). To obtain positions for another time, t, people need to apply the formula

$$x(t) = x(1997.0) + vx \cdot (t - 1997.0) \quad (2)$$

and similar formulas for y(t) and z(t). Here, x(t) denotes the point's x-coordinate at time t, x(1997.0) denotes the point's x-coordinate on 1 January 1997, and vx denotes the x-component of the point's velocity.

NGS furnishes easy access to the ITRF96 reference frame through a set of over 170 stations belonging to the National CORS network (recall Fig. 2). Positions, velocities, and other pertinent information for these stations are available via the Internet by accessing: *ftp://www.ngs.noaa.gov/cors/coord/coord_96*.

Transforming Between Reference Frames

In 1998, U.S. and Canadian officials jointly adopted a Helmert transformation to convert positional coordinates between ITRF96 and NAD 83 (CORS96). The IERS has also adopted appropriate Helmert transformations for converting between ITRF96 and other ITRS realizations. NGS has encoded all these transformations into a software package, called HTDP (Horizontal Time-Dependent Positioning), which is freely available via the Internet: *http://www.ngs.noaa.gov/TOOLS/Htdp/Htdp.html*

This software enables people to transform individual positions entered interactively or a collection of positions entered as a formatted file. Also, if people expect to transform only a few positions, then they may run HTDP interactively from this web page.

While Helmert transformations, as encoded into HTDP, are appropriate for transforming positions between any two ITRS realizations or between any ITRS realization and NAD 83 (CORS96), more complicated transformations are required for conversions that involve NAD 27, NAD 83 (1986), or NAD 83 (HARN). These complications arise because these frames contain large local and regional distortions that can not be quantified by a simple Helmert transformation. For instance, NAD 27 contains distortions at the 10 m level. That is, if one applied the best possible Helmert transformation from NAD 27 to NAD 83 (CORS96), then the converted NAD 27 positions may still be in error by as much as 10 m. In a similar manner, NAD 83 (1986) contains distortions at the 1 m level, and NAD 83 (HARN) contains distortions at the 0.1 m level.

NGS has developed a software package, called NADCON (), that embodies rather intricate transformations to convert positional coordinates between any pair of the following reference frames: NAD 27, NAD 83 (1986), and NAD 83 (HARN). Referring to a pair of 2D grids that span the United States, NADCON contains appropriate values for each grid node to transform its positional coordinates from one reference frame to another. Furthermore, NADCON interpolates these gridded values to transform points located within the grid's span. It should be noted that NADCON may be used only to transform horizontal coordinates (latitude and longitude), because ellipsoidal heights—relative to NAD 27 or NAD 83 (1986)—have never been adopted for most control points.

While HTDP may be used with pairs of certain reference frames (NAD 83 (CORS96), ITRF88, ITRF89,. . . , and ITRF97) and NADCON with pairs of other reference frames (NAD 27, NAD 83 (1986), and NAD 83 (HARN)), no NGS-sanctioned software exists for transforming coordinates from any member of one set to any member of the other. Also, no NGS-sanctioned software exists for transforming NAD 83 (CORS93) and/or NAD 83 (CORS94) positions to other reference frames. Regarding the WGS 84 reference system, it is generally assumed that WGS 84 (original) is identical to NAD 83 (1986), that WGS 84 (G730) is identical to ITRF92, and that WGS 84 (G873) is identical to ITRF96. Other transformations between pairs of the WGS 84 realizations, however, have also appeared in the literature.

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Modern Terrestrial Reference Systems—Part 4:

Practical Considerations for Accurate Positioning

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Prior to the space-age, people primarily used terrestrial techniques like triangulation and/or trilateration to position points. In those days, they had essentially only one choice for a nationwide reference frame; namely, NAD 27. To position new points, a person would measure distances and angles that related the unknown positional coordinates of the new points to the known coordinates of some pre-existing terrestrial control points. More often than not, these known coordinates were referred to NAD 27. Hence, calculated positions for the new points were automatically referred to NAD 27.

In the 1970s, people started using Earth-orbiting satellites for positioning points. In particular, before GPS became operational, a number of people used the Doppler satellite system, known as TRANSIT, to position points with an accuracy of several meters relative to the geocenter. Now people use GPS to position points even more accurately. Hence, satellites have become flying control points whose time-dependent positions are well known thanks to diligent work by several institutions. Currently, GPS satellite orbits are available in appropriate realizations of both WGS 84 and ITRS, but not in any NAD 83 realization.

Confusion Can Be Introduced

Techniques that position new points by using only satellites as control automatically yield positional coordinates that are referred to the same reference frame as that used for the orbits, either some realization of WGS 84 or some realization of ITRS. Satellite-only positioning techniques, however, do not currently enable the centimeter-level positioning accuracy that is possible with techniques, like the static differential carrier phase GPS technique, that involve using both satellite and pre-existing terrestrial points as control. The use of an integrated approach, however, introduces confusion when the satellite orbits are referred to one reference frame and the terrestrial control points are referred to another, as the newly computed positions will be referred to some ambiguous hybrid reference frame. Although this hybrid reference frame will approximate the reference frame of the terrestrial control points, the error of this approximation at a given point depends on many factors including the distances and directions from this point to the various terrestrial control points being used as well as the magnitude and orientation of the vector connecting the origins of the two reference frames. For the case of a single terrestrial control point referred to NAD 83 with the GPS orbits referred to ITRF96, the error of this approximation at a point will grow less than 1 cm for every 100 km of distance to the terrestrial control point. Hence, for accurate positioning, people should use the same reference frame for both the orbits and the terrestrial control points, especially if the distance to the nearest terrestrial control point is large. If for some reason the available orbits are expressed in one reference frame while the terrestrial control is expressed in another, then the positions of the terrestrial control points should ideally be transformed into the given reference frame of the orbits, as may be accomplished, for example, by using the HTDP software. Alternatively, one could transform the orbits to the reference frame of the terrestrial control points, but this option is likely to involve more computations. Moreover, the software being used to process GPS data may require the orbits to be expressed in a realization of a specific reference system. Thus, it is usually better to transform the terrestrial control points rather than to transform the orbits.

Now, once the orbits and the terrestrial control are expressed in a common reference frame, the computed coordi-

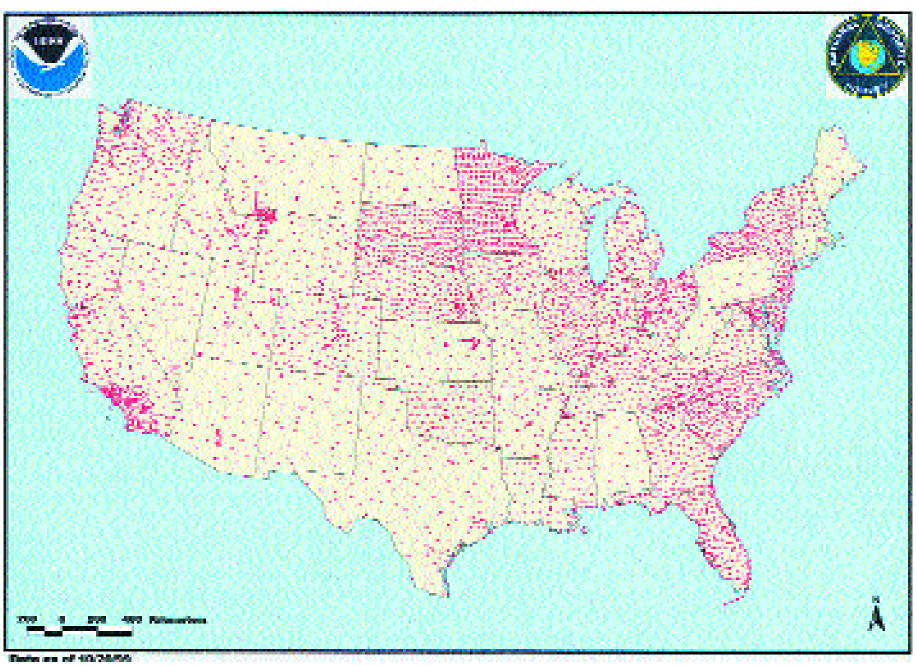

Figure 5: GPS stations defining the National Spatial Reference System: NAD 83 (NSRS)

nates of the newly positioned points will also be referred to this reference frame. Hence, if people need these positional coordinates to be expressed relative to some other reference frame, then they will need to transform these coordinates. Again, the HTDP software will serve this purpose.

For rigorous computations, the epoch date of the positional coordinates of the terrestrial control should agree with the date that the GPS observations were performed. This agreement is especially important when computations are performed in an ITRS realization because terrestrial points are moving relative to this reference system. As previously mentioned, ITRF96 velocities have magnitudes ranging between 10 and 20 mm/yr in the coterminous United States, and these magnitudes are even higher in Alaska and Hawaii. If velocities at the control points are known, then equation (2) may be applied to convert the positional coordinates of these points from their given epoch date to the date that the GPS observations were performed. If velocities of these control points are unknown, then these velocities may be predicted using the HTDP software.

Use of Precise Orbits Recommended

As previously implied, we are recommending the use of highly precise postfit GPS orbits expressed in ITRS coordinates for accurate GPS positioning. The International GPS Service (IGS) freely distributes such orbits through their website *http://igscb.jpl.nasa.gov/*. The IGS orbits are the result of computational efforts involving seven different institutions located in four different countries: Canada, Germany, Switzerland, and the United States. Three of these seven institutions are U.S. based: the Jet Propulsion Laboratory, Scripps Institute of Oceanography, and NGS. Each of the seven institutions computes GPS orbits essentially independently, and then IGS rigorously combines the seven solutions to produce the orbits that they distribute. Also, the NGS-derived postfit orbits are available at *www.ngs.noaa.gov/GPS/GPS.html*

Finally for accurate positioning, sites from the National CORS system provide good terrestrial control. These CORS sites have been positioned with great accuracy, as they form the foundation of the National Spatial Reference System. Each site contains a geodetic-quality GPS receiver whose carrier phase and code range measurements are transmitted to NGS headquarters in Maryland. NGS then makes these data freely available via the Internet. In particular, people may download CORS data either via anonymous FTP (file transfer protocol) using the address *ftp://www.ngs.noaa.gov/cors/* or through the World Wide Web using the address *http://www.ngs.noaa.gov/CORS/*. Furthermore, people may pose CORS-related questions to NGS via the email address *cors@ngs.noaa.gov*. Whether using CORS or other sites as terrestrial control, it is usually best to include several points for the sake of redundancy and to suppress those systematic errors that grow as a function of distance.

The Species of "Origins"

In 1859, Charles Darwin published his revolutionary book, *The Origin of Species*, in which he documented evidence for the theory of how various forms of life evolve to better adapt to their environment. Inasmuch as this theory of evolution applies to humans, it may also apply to the products that humans develop. In particular, the theory of evolution may apply in large measure to terrestrial reference systems, as these human artifacts evolve to adapt to our knowledge of our environment and to become commensurate with our ability to position points with ever increasing accuracy.

In this series of articles, we have discussed how three particular species of reference systems—NAD 83, WGS 84, and ITRS—have evolved over the past couple of decades. Surely, they will continue to evolve into the future. Indeed, U.S. and Canadian representatives are already planning to introduce a new realization of NAD 83. The U.S. geospatial community will come to know this new realization as NAD 83 (NSRS) where NSRS is an acronym for National Spatial Reference System. NAD 83 (NSRS) will serve to supercede the conglomeration of regional reference frames that comprise NAD 83 (HARN). In particular, NAD 83 (NSRS) will incorporate newly computed positional coordinates for all HARN control points, as well as other control points, so that these new positions will be consistent with adopted CORS positions (Figure 5). Currently, horizontal discrepancies as large as 7 cm exist between NAD 83 (HARN) positions of control points and their idealized NAD 83 (CORS96) positions. Significantly greater discrepancies exist in the vertical dimension, because the accuracy of ellipsoidal heights measured during the earlier HARN surveys compares poorly relative to today's height-measuring capability. Consequently to obtain more accurate heights, NGS is cooperating with several organizations to resurvey the HARN in each State. These HARN resurveys reflect NGS's Height Modernization Initiative to support the public's growing use of GPS to measure accurate heights. NGS expects to complete the HARN resurveys around 2002. Moreover, NGS will wait until these resurveys have been completed before releasing NAD 83 (NSRS), as this agency plans to combine these new observations rigorously with existing observations to compute accurate ellipsoidal heights that are consistent across the country. NGS is also developing more and more accurate geoid models to convert such ellipsoidal heights to appropriate orthometric heights. Additional details about this forthcoming NAD 83 realization may be found at the Internet address: *http://www.ngs.noaa.gov/initiatives/new_reference.shtml.*

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Transformation Parameters from ITRF96 to NAD83 (CORS96)

In the summer of 1998, NGS and Canada's Geodetic Survey Division adopted the following parameters for transforming positions in North America from ITRF96 to NAD 83 (CORS96) via equation (1) :

For the three shifts:

Tx = 0.9910 m
Ty = -1.9072 m
Tz = -0.5129 m.

For the scale factor:

s = 0.0 (unitless).

For the three rotations (counterclockwise sense positive):

Rx = [25.79 + 0.0532 • (t - 1997.0)]• mr radians
Ry = [9.65 - 0.7423 • (t -1997.0)]• mr radians
Rz = [11.66 - 0.0316 • (t - 1997.0)]• mr radians.

In the above equations, mr = 4848.13681• (10**-12) is the conversion factor from milli-arcseconds to radians; t is the "epoch date" in years (e.g., 1999.3096 0h UTC, 23 April 1999). Recall that the epoch date is the date to which the given positions correspond. Note that the rotations, Rx, Ry, and Rz, are time dependent because the North American tectonic plate moves relative to ITRF96 while this plate is essentially stable relative to NAD 83 (CORS96).

Because ITRF94 and ITRF97 have the same origin, orientation, and scale as ITRF96, these same parameters are applicable for transforming these other two ITRS realizations to NAD83 (CORS96). On the other hand, these parameters are not applicable for transforming ITRF93 and earlier ITRS realizations to NAD 83 (CORS96).